

## A Critical Examination of Ibn-Sina's Theory of the Conditional Syllogism

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### **Abstract:**

This paper will examine Ibn Sina's theory of the Conditional Syllogism from a purely logical point of view, and will lay bare the principles he adopted for founding his theory, and the reason why the newly introduced part of his logic remained undeveloped and eventually was removed from the texts of logic in the later Islamic tradition.

As a preliminary discussion, this paper briefly examines Ibn Sina's methodology and gives a short summary of the relevant principles of Aristotelian logic, before delving into the analysis of Ibn Sina's treatment of the conditional, which is the heart of the paper. This analysis explains Ibn Sina's theory of conditionals in systematic stages, explaining his motivation at each step and showing the weaknesses in his argument using the tools of modern symbolic logic. The paper concludes by mentioning a few of Ibn Sina's remarkable insights regarding conditionals.

Key terms: Conditional Syllogism, Ibn Sina, Aristotelian Logic.

### **Introduction**

After the publication of Ibn-Sina's *al-Shifa: al-Qiyas*, edited by S. Zayed, Cairo, 1964, where Ibn-Sina presented most extensively his theory of the

conditional syllogism and, later on, the publication of Nabil Shehaby's translation of it into English in 1973<sup>1</sup> I think we have all we should have in hand to evaluate Ibn-Sina's theory as it is, a theory Ibn-Sina regards as his important contribution to Aristotelian logic and as a new form of argument "unknown until now, which I myself discovered"<sup>2</sup> (my translation).

My task in this paper is limited. I am going to examine Ibn-Sina's theory from a purely logical point of view, and to lay bare the principles he adopted for founding his theory, and the reason why the newly introduced part of his logic remained undeveloped and eventually was removed from the texts of logic in the later Islamic tradition.

Anyone interested in the long and controversial history of conditional syllogisms or philosophical, theological or dialectical motivations of the subject should consult the growing literature of it, now easily available. Here, I only mention Nicholas Rescher's paper: *Avicenna on the logic of "conditional proposition"*, published in 1963<sup>3</sup>. Perhaps that was the first pioneering paper on the subject in the English language. But at the time of writing his paper the text of al-Qiyas had not been published. So Rescher wisely remarked: "until it is available, the present discussion must be viewed as tentative"<sup>4</sup>. Rescher's paper is descriptive. He also recognizes some invalid arguments in the theory without finding out the methodological reasons for those invalidities. Nabil Shehaby's introduction to his translation is also purely descriptive, though informative. In this paper I quote Ibn-Sina's views in al-Qiyas from this translation which provides readers with references to the pages and lines of the Cairo edition mentioned above.

### **Preliminaries**

Aristotelian logic is also called "term logic". By term, here, is meant concept – term which stands as the subject or predicate of a categorical proposition. In modern logic concept –terms are treated as one-place predicates. So in modern terminology one might say that the Aristotelian logic is a monadic logic. But this is highly misleading. By monadic logic, in a modern sense, we

mean one-place predicate logic based on propositional logic as its fundamental part. But in the Aristotelian logic this part is missing. So Ibn-Sina's name for his theory as "theory of conditional syllogisms" is more appropriate than "theory of propositional logic". In fact he never uses letters standing for proposition. He always uses subject-predicate forms with letters standing only for concepts, and never writes "If P, then Q" but always writes "If A is B, then C is D", with or without a quantifier for the antecedent or consequent. Of course Ibn-Sina's theory is meant to have the same status in the Aristotelian logic that propositional logic has in modern logic. But they are formally worlds apart. They are founded on a quite different, in fact opposite principle. And this is where the question of methodology, not properly discussed yet, arises.

Ibn-Sina works within the frame-work of Aristotle's logic, which is based on the theory of categorical syllogisms. In a famous passage in the *Prior Analytic*, Aristotle wrote:

"Many other conclusions also are reached by hypothesis, and these require further study and clear explanation. What their differences are, and in how many ways a hypothetical conclusion is effected, will be described later for the present let us regard this much as evident: that it is impossible to analyse such syllogisms as these into the figures."<sup>5</sup>

But this promise was never carried out, and, more surprisingly, his anticipation that: "it is impossible to analyse such syllogisms as these into the figures" was never taken seriously by his followers, notably Ibn-Sina. It is, however, to his credit that he realized the importance of conditional syllogisms more than many other logicians and in al-Qiyas wrote: "Many theses in mathematics, physics, and metaphysics are connective (*muttasila*) or separative (*munfasila*) conditional<sup>6</sup>, meaning by "connective", implication or chance conditional and by "seperative", disjunctive combination.

### **Ibn-Sina's Methodology**

Ibn-Sina's overall methodology is to establish a parallelism or correspondence between the conditional and categorical syllogisms, in fact a reduction of the former to the latter. When this is done he can claim, in particular in his shorter books and treatises (of which more than 30 authentic ones are recorded), that:

"You must treat the connective conditionals in a quantified form or indefiniteness, contradiction and conversation as you treat categorical with the antecedent as a subject and the consequent as a predicate"<sup>7</sup> (my translation).

Within this theory the validity of a simple sequent like:

$$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$$

must be given according to the rules of the categorical syllogisms. But before subjecting such sequent to those rules he has to cloth them in the forms resembling categorical propositions. Before examining Ibn-Sina's theory a short reminder of the theory of categorical syllogisms is in order.

### **A short summary of the principles of Aristotelian logic**

Ibn-Sina's theory is based on principles among which the following are of immediate interest for my discussion:

- 1- A predicative sentence consists of two main parts: subject-term and predicate-term. The third part is copula determining the quality of the sentence.
- 2- There are four types of predicative sentences: universal affirmative (A), universal negative (E), particular affirmative (I), and particular negative (O). As to the singular statements it is safe to say that within this theory they have not received proper treatment (this has its own history in which I am not interested here).
- 3- Inferences are of two types: immediate and syllogistic.

- a. There are different kinds of immediate inferences of which I only mention the following two principles: form "All A is B" follows both "some B is A" (conversion per accidens) and "some A is B" (simple conversions).
- b. Categorical syllogism. A categorical syllogism has three sentences, two as premises and one as conclusion. The two premises must have a term in common (middle term). This term, which connects the two premises, does not appear in the conclusion. The terms standing as subject and predicate in the conclusion are, respectively, called minor and major term. The premise containing the major term is called the major premise and the one containing the minor term, the minor premise. The middle term may be the subject in both premises, or the predicate of both, or the subject only of the minor or the subject only of the major premise. So we have four figures. Aristotle discusses only three figures as does Ibn-Sina, who mentions briefly the fourth figure, in which the middle term is the subject of the minor premise and a predicate of the major, and discards it.

The Theory of syllogism is a set of rules prescribing which of all possible forms (moods) of each figure are valid.

Now the fundamental methodology of Ibn-Sina is to embed any inference of hypothetical syllogisms within the frame-work of the theory of categorical syllogisms. If we lose sight of this point we will be bound to read many irrelevant interpretations into it. In this paper I shall confine my discussion to Ibn-Sina's analysis of the connective conditional whose truth-conditions are exactly the same as the material conditional in the modern sense, i.e. a conditional which is false if and only if the antecedent is true and the consequent false. But Ibn-Sina's understanding of this conditional is a kind of implication where the consequent is related and follows somehow from the

antecedent. He distinguishes this one from the chance conditional with truth-conditions totally different from the connective conditional. It is in the case of the latter that Ibn-Sina's methodology can be seen clearly in application.

### **Ibn-Sina's analysis of conditional**

In this part I shall try, through systematic stages, to explain Ibn-Sina's motivation at every step in reducing conditionals to what I would like to call pseudo-categorical propositions.

- 1) The first major difficulty is that in "If P then Q" both "P" and "Q" stand for proposition. How can a conditional consisting of two propositions be reduced to a single seemingly categorical one? Ibn-Sina's way out of this difficulty is to deny that the antecedent and the consequent of a conditional are sentences (propositions). His interpretation of "If it is so, then it is so" is as follows:

"When you say 'If it is so' it is neither true nor false; and when you say 'then it is so' it is also neither true nor false provided that 'then' fulfils its real function of indicating that something follows from another."<sup>8</sup>

This argument can lead only to one conclusion: a conditional as a whole is one proposition. Then after some conflicting remarks, Ibn-Sina concludes that in "if P, then Q", "P" and "Q" play the same role respectively that subject- term and predicate-term play in a categorical proposition.

- 2) Now the second difficulty arises. In an inference each premise must be one of A, E, I, and O. Therefore to reduce 'If P, then Q', to categorical forms we need to introduce quantifiers. This is a critical point which may easily give rise to the misinterpretation of the nature of these quantifiers. Let me explain why.
- 3) In a sentence like:

A triangle is a shape  
one can easily introduce a quantifier:  
every/some triangle is a shape

But in a conditional like:  
If the sun rises, then it is day  
it is just meaningless to say:  
every/some if the sun rises, then it is day

The reason is obvious. In 'if P, then Q', "P" and "Q" are not concepts. There is no extension here over which quantifiers may range. Here, however, another kind of expression can be used:

"Always/ under any condition if the sun rises, then it is day."

It is not the case that Ibn-Sina could have used ordinary quantifiers but he chose not to use them and used another kind of quantifier. On the other hand, these expressions need not be of temporal nature. All Moslems logicians are in agreement with Ibn-Sina that:

"In the statement 'Always: when C is B, then H is Z' the words 'Always: when' are not only meant to generalize the occurrences of the statement, as if one said: "Every time C is B, then H is Z", but then are also meant to generalize the conditions which we may add to the sentence 'C is B' for the antecedent may refer to something which does not recur and is not repetitive.<sup>9</sup>"

To emphasize that these expressions are not necessarily of a temporal nature Ibn-Sina discusses conditionals expressing chance connection. Then in giving the truth-conditions of:

Always: when man talks, then the donkey brays.

Imaging a certain time at which no donkey exists, he writes:

"It might be thought that at this specific time....the proposition 'always: when man talks,

then the donkey brays' is false. For at this time there are no donkeys to bray. But this is a false opinion. For the statement 'every donkey brays' is true even if there are no donkeys to bray."<sup>10</sup>

This clearly shows that these expressions are not meant to be only temporal. That is why, I think, it is a mistake to use temporal operators and translate the conditional mentioned above into:

$$\forall_t (R_t (P) \rightarrow R_t (Q))$$

with interpreting "R<sub>t</sub>P" as "realization of P at the time t".

Hereafter I shall call these expressions as pseudo-quantifiers, and show

them by  $\forall_s$  and  $\exists_s$ . "s" is a variable ranging over any situation temporal or

otherwise.

3) The last step is to impose four types on these pseudo-quantified conditionals corresponding to the four types of categorical propositions. Without going into further details, and based on my close examinations of Ibn-Sina's writings as well as the writings of the later Moslem logicians, the following formalization of the types of conditionals suggests itself:

$$AC: \forall_s (P_s \rightarrow Q_s)$$

$$\text{EC: } \forall_s (P_s \rightarrow \sim Q_s)$$

$$\text{IC: } \exists_s (P_s \& Q_s)$$

$$\text{OC: } \exists_s (P_s \& \sim Q_s)$$

Now by establishing this parallelism between the conditionals and categorical propositions, Ibn-Sina, as quoted before, claims that all rules of inferences applicable to the categorical are equally applicable to conditional. Now let us examine some cases where Ibn-Sina applies his theory.

**a- Conversion simpliciter**

In the conversion simpliciter the antecedent is turned into consequent and the consequent into an antecedent, while keeping the quality and truth unchanged. This is Ibn-Sina's first example:

From stating that 'Never: when every A is B, then every C is D' it evidently follows that: Never: when every C is D, then every A is B<sup>11</sup>

In symbolism:

$$\text{From } \forall_s (P \rightarrow \sim Q) \quad \text{follows } \forall_s (Q \rightarrow \sim P)$$

This is parallel to the *conversion simpliciter* of "No A is B" which is "No B is A"

In this theory this inference holds and Ibn-Sina's proof of it is valid.

Now let us apply the same rule to a universal affirmative. Here from "Always: when every A is B, then every C is D" we get by the rule corresponding to the universal categorical: "sometimes: when every C is D,

then every A is B" or from " $\forall_s (P_s \rightarrow Q_s)$ ", we get " $\exists_s (P_s \& Q_s)$ ". Now here

parallelism fails. Although from "Every A is B", given the existential import of the subject we can get: "some B is A", but it does not apply to "P" as a sentence. In fact one consequent of this rule is the following:

From " $\forall_s (P_s \& \sim P_s \rightarrow Q_s)$ " follows " $\exists_s (P_s \& \sim P_s) \& Q_s$ ", which is obviously

invalid.

### b- Syllogism

More revealing is Ibn-Sina's proof of the third mood of the third figure of conditional syllogisms. Here I quote him in detail:

"This mood is compounded of two universal affirmative propositions

always: when C is D, then H is Z;

and

always: when C is D, then A is B

therefore

sometimes: when H is Z, then A is B"

Then, by reductio, he gives the following proof:

"Let (the conclusion) be

'Never: if H is Z, then A is B'

If we add to it:

'always: when C is D, then A is B

both will yield the following conclusion:

'Never: if C is D, then A is B'

This is contradiction"<sup>12</sup>

This proof is carried out in the same way that the proof of its corresponding mood of the categorical syllogism:

Every A is B

Every A is C

and given the existential presupposition that "some A exists":

we have:

Some B is C

Now for comparison, and to see it clearly, Ibn-Sina's proof in symbolism is as following:

$$\forall_s (P_s \rightarrow Q_s)$$

$$\forall_s (P_s \rightarrow R_s)$$

therefore,

$$\exists_s (Q_s \& R_s).$$

Now by reductio,

$$\sim \exists_s (Q_s \& R_s)$$

or

$$\forall_s (Q_s \rightarrow \sim R_s)$$

from this and the first premise we get

$$\forall_s (P_s \rightarrow \sim R_s)$$

Now Ibn-Sina claims this conditional is contradictory to the second premise, I.e.

$$\forall_s (P_s \rightarrow R_s)$$

This is how he understands the negation of his quantified conditional. Rescher believes that by so doing: "He has, in effect, broadened the categories of "conjunctive" and "disjunctive" propositions beyond their original characterization"<sup>13</sup>. Rescher, I believe, fails to note the reductive nature of Ibn-Sina's quantifiers used for conditionals, a reduction which is supposed to reduce every quantified conditional to the corresponding categorical proposition. The proof under discussion is defective for three reasons:

1. Propositions are not concepts with extensions and so not obtainable by existential import;
  
2.  $\forall_s(P_s \rightarrow R_s)$  and  $\forall_s(P_s \rightarrow \sim R_s)$  are not contradictory;
  
  
3.  $\exists_s(Q_s \& R_s)$  is not a consequence of  $\forall_s(P_s \rightarrow Q_s)$  and  $\forall_s(P_s \rightarrow R_s)$ .

So here reductio ad absurdum has no useful application.

All this shows the limitations inherent in the Aristotelian syllogism as the building blocks of propositional logic. In fact I think that Ibn-Sina's theory suffers from violating a principle so fundamental to all sciences and in particular to logic and mathematics: the principle of structuring the complex out of the simples. As Lukasrewiez rightly observes even in the limited theory of Aristotle's syllogism, Aristotle had to use theses of propositional logic "to reduce syllogisms of the second and third figures to the syllogisms of the first figure".<sup>14</sup>

Propositional logic, as Frege shows us, is the most simple and fundamental part of logic upon which more complex and complicated logics should be founded. But Ibn-Sina's theory is exactly the other way round. I examined only some simple cases of the application of his theory. When we come to his more complicated conditional syllogisms many inferences become so involved and lead to invalid syllogisms. No wonder that logicians following Ibn-Sina found the theory so difficult and confusing that eventually regarded it as dispensable in practice and not worthy of serious consideration.

Whether Ibn-Sina's theory can be saved by introducing ontology of situations or a kind of the Davidsonian ontology of events for the quantified

conditionals corresponding to existential import for the categorical propositions would be a matter of further research, which I am not pursuing here. But if that could be done, many of the invalid inferences, including some mentioned so far, would be turned into valid ones.

Putting, however, your finger on short comings of a work of a past master who lived more than one thousand years ago and judging his theory from modern point of view without mentioning his great innovations and ingenious insights into the subject is certainly unfair. I would like to end by mentioning briefly only a few of Ibn-Sina's many remarkable insights on the conditionals:

1. Ibn-Sina is quite aware of the differences between conditionals and categorical propositions and the impossibility of reducing the former to the latter generally. So he writes:

“The person who thought that the proposition: ‘Always: when A is B, then H is Z’ is predicative because ‘Always: when this is a man, then he is an animal’ is equal to ‘Every man is an animal’ is mistaken for the following reasons”<sup>15</sup>

Ibn-Sina's reasons are best summarized in one of his shorter books as follows:

A difference between the antecedent and the consequent, on the one hand, and the subject and the predicate, on the other hand, is that the subject and the predicate can be single terms, but the antecedent and the consequent can never be... Another difference between the antecedent and the consequent of the conditional, and the subject and the predicate of the categorical is that it is possible to ask about a subject predicate proposition whether or not the predicate belongs to the subject. For example when someone says

“Zia is alive” you may ask whether he is or he is not. But when someone utters a conditional you cannot ask whether or not the consequent belongs to the antecedent<sup>16</sup>.

2. Ibn-Sina realizes that some conditionals are in fact equivalent to some categoricals. So he distinguishes what is now called general conditionals from material conditionals:

[T]he connective in which the antecedent and consequent share one part can be reduced to predicative propositions – as when you say, for example, “If a straight line falling on two straight lines make the angle on the same side such and such, the two straight lines are parallel”. This is equivalent in force (fi quwwati) to the predicative proposition: “Every two straight lines on which another straight line falls in a certain way are parallel”<sup>17</sup>

3. Ibn-Sina’s classifications of connective and separative (disjunctive) conditionals, various combinations and his truth-functionally treatment of them, within the limitations of Aristotelian logic is, perhaps, unprecedented. Thus Rescher writes:

[A] fully articulated theory of logic of hypothetical and disjunctive proposition is apparently first to be found in the logic treatises of Avicenna<sup>18</sup>.

**Conclusion:**

Ibn-Sina’s theory of hypothetical syllogisms is supposed to be the missing part of Aristotle’s logic. Ibn-Sina, by introducing quality and quantity to the conditional, tries to reduce each conditional to a

form corresponding to its corresponding categorical in order to apply the rule of inferences applicable to the categorical equally applicable to the conditional. But the parallels between the two logics break down. Whether by introducing a kind of Davidsonian ontology for situations or events and providing it with an existential import we could save Ibn-Sina's theory of invalid consequences remains to be seen.

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